Semper FI?

Supercurrents, R-symmetries, and the Status of Fayet-Iliopoulos Terms in Supergravity

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Based on work done with Keith Dienes
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The FI Term

[Fayet & Iliopoulos, ’74; Fayet, ’75]

- The FI term lies at the heart of D-term SUSY-breaking, one of the two paradigms for how SUSY can be broken. In a theory with an Abelian factor, one can write:

$$\boxed{\mathcal{S}_{\mathrm{FI}} = \int d^4x d^4\theta \; 2\underset{\text{FI parameter}}{\xi} V = \int d^4x \xi \left(D + \underbrace{\frac{1}{2}\Box C}_{\text{Total derivative}} \right)}$$

- At the level of the action, it is both gauge and SUSY invariant.

- The FI term has become one of the standard items in the SUSY model-builder’s toolbox.

However, FI terms have some unusual properties:

- They have very restrictive renormalization-group flows.

- They almost never dominate in dynamical SUSY breaking.

A Tortu(r)ous History

- The first steps toward embedding FI terms in supergravity were taken in [Freedman, '78].
- This picture was refined in [Stelle & West, '78], where it was shown that FI terms in SUGRA lead to invariance under a peculiar linear combination of super-Weyl shifts and gauge transformations known as a "gauged R-symmetry."
- In [Barbieri et al., '82], it was shown that such theories could only be coupled to matter in theories which possess a global R-symmery.
- Studies of the gauge-anomaly structure of theories with a gauged R-symmetry [Chamseddine & Dreiner, '95; Castano et al. '96; Bineutry et al., '04; Elvang et al., '06; many others].
- It has also been shown [Witten, '86] that this setup leads to problems with Dirac quantization.

A Tortu(r)ous History

- Recently, it has also been shown [Seiberg & Komargodski, '09] that this construction gives rise to gauge-non-invariant supercurrents and results in additional global symmetries in the full SUGRA theory, in conflict with commonly held beliefs about additional global symmetries in supergravity.
- This has led to speculation that FI terms may be completely ruled out in supergravity theories.

Clearly, this 30-year saga indicates that the issues involved in coupling theories with FI terms to supergravity are numerous and quite subtle.

Outline of the Talk

- Symmetry currents in SUSY theories and their multiplet structure: A review.
- The strange case of the FI contribution: Incomplete multiplets and R_5-symmetry issues.
- Supergravities and compensator formalisms: basic facts.
- A comparison of FI terms in two different supergravity formalisms, which are distinguished by their R_5-symmetry properties:

  1 The chiral (or "old minimal") formalism
  (which explicitly breaks local R_5-invariance)

  2 The linear (or "new minimal") formalism
  (which manifestly preserves local R_5-invariance)

- Observations and conclusions.

The Supercurrent Supermultiplet

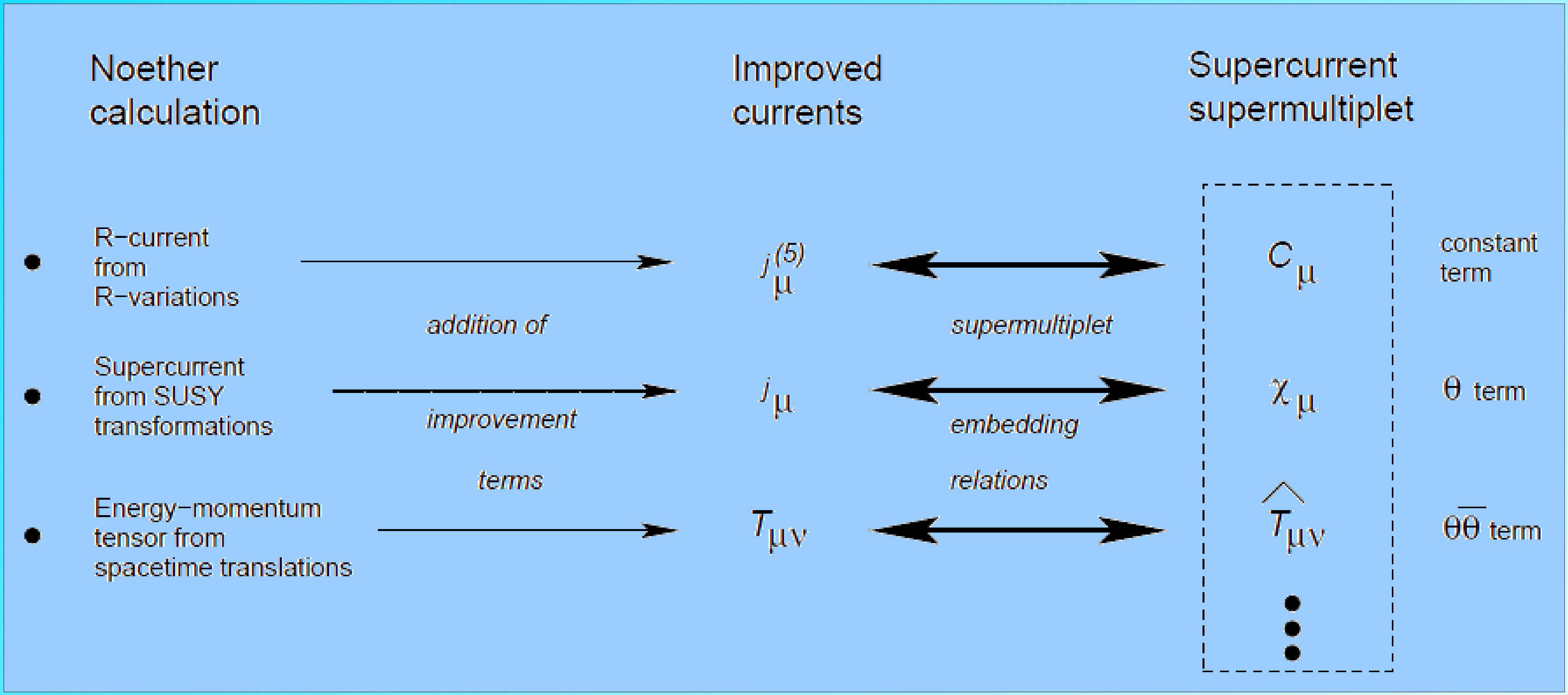
- To make a globally supersymmetric theory local, one must be able to couple the symmetry currents for SUSY transformations ($j_{\mu\alpha}$) and spacetime translations ($T_{\mu\nu}$) to the connection fields $\psi_{\mu\alpha}$ and $g_{\mu\nu}$ of supergravity.
- These currents, along with a particular R-symmetry current (commonly called the R_5-current) can be embedded into a real vector supermultiplet $J_{\alpha\dot{\alpha}} = \sigma^{\mu}_{\alpha\dot{\alpha}} J_{\mu}$ [Ferrara & Zumino, '75].

The supercurrent supermultiplet

The supercurrent supermultiplet in component form:

$$
\begin{aligned}
J_\mu \quad = \quad & C_\mu + i\theta\chi_\mu - i\overline{\theta}\overline{\chi}_\mu + \frac{i}{2}\theta\theta(M_\mu + iN_\mu) - \frac{i}{2}\overline{\theta\theta}(M_\mu - iN_\mu) - \theta\sigma^\nu\overline{\theta}\hat{T}_{\nu\mu} \\
& + i\theta\theta\overline{\theta}\left(\overline{\lambda}_\mu + \frac{i}{2}\overline{\sigma}^\nu\partial_\nu\chi_\mu\right) - i\overline{\theta\theta}\theta\left(\lambda_\mu + \frac{i}{2}\sigma^\nu\partial_\nu\overline{\chi}_\mu\right) + \frac{1}{2}\theta\theta\overline{\theta\theta}\left(D_\mu + \frac{1}{2}\Box C_\mu\right)
\end{aligned}
$$

- The relationship between the Noether currents and the lowest components of $J_{\alpha\dot{\alpha}}$ is highly nontrivial.



- In general, $j_{\mu}^{(5)}$, $j_{\mu\alpha}$, and $T_{\mu\nu}$ must be modified by the addition of improvement terms, then embedded (often in a complicated way) into C_{μ}, $\chi_{\mu\alpha}$, and $\hat{T}_{\mu\nu}$

Anomalies and Conservation Laws

- In superconformal theories, the conservation laws for $j_\mu^{(5)}$, $j_{\mu\alpha}$, and $T_{\mu\nu}$ can be written as a superfield-level conservation law for $J_{\alpha\dot\alpha}$:

$$\boxed{\overline{D}^{\dot\alpha} J_{\alpha\dot\alpha} = 0 \qquad \text{(superconformal)}}$$

- In theories with broken superconformal invariance, this conservation equation is modified by the addition of superconformal anomaly terms on the R.H.S.

Possible Superconformal Anomalies:

$\partial_\mu j^{\mu 5} \neq 0$ ⟵ R_5-symmetry current not conserved

$T^\mu_\mu \neq 0$ ⟵ Stress-energy tensor not traceless

$(\overline{\sigma}^{\mu\dot\alpha\alpha} j_{\mu\alpha}) \neq 0$ ⟵ Supercurrent not “traceless” under $\overline{\sigma}^{\mu\dot\alpha\alpha}$ contraction

- Several different anomaly structures are possible, and each leads to a different conservation law for $J_{\alpha\dot{\alpha}}$ …

$$T^\mu_\mu \neq 0,\ (\overline{\sigma}^{\mu\dot{\alpha}\alpha} j_{\mu\alpha}) \neq 0,\ \partial_\mu j^{\mu 5} \neq 0 \longrightarrow \overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S$$

Chiral multiplet:
$(S = \overline{D}^2 T_S)$

$$T^\mu_\mu \neq 0,\ (\overline{\sigma}^{\mu\dot{\alpha}\alpha} j_{\mu\alpha}) \neq 0,\ \partial_\mu j^{\mu 5} = 0 \longrightarrow \overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = L_\alpha$$

Field-strength multiplet:
$(L_\alpha = \overline{D}^2 D_\alpha T_L)$

…and a different embedding of $j^{\mu 5}$, $j^{\mu\alpha}$, and $T^{\mu\nu}$ into the components of $J_{\alpha\dot{\alpha}}$:

Superconformal and Linear cases:
$$\begin{cases} j_{\mu\alpha} = \chi_{\mu\alpha} \\ T_{\mu\nu} = -\frac{1}{4}(\hat{T}_{\mu\nu} + \hat{T}_{\nu\mu}) \end{cases}$$

Chiral case:
$$\begin{cases} j_{\mu\alpha} = \chi_{\mu\alpha} + (\sigma_\mu \overline{\sigma}^\nu \chi_\nu)_\alpha \\ T_{\mu\nu} = -\frac{1}{4}(\hat{T}_{\mu\nu} + \hat{T}_{\nu\mu} - 2 g_{\mu\nu} \hat{T}) \end{cases}$$

Example: U(1) Gauge Theory Without an FI Term

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}|_{\overline{\theta\theta}}\right)\\ &= -\frac{1}{4}F^2 - \frac{i}{2}\lambda\sigma^\mu(\partial_\mu\overline{\lambda}) + \frac{i}{2}(\partial_\mu\lambda)\sigma^\mu\overline{\lambda} + \frac{1}{2}D^2\end{aligned}$$

- This leads to the equations of motion ...

$$\partial_\mu F^{\mu\nu} = 0\ , \qquad (\overline{\sigma}^\mu\partial_\mu\lambda)^{\dot{\alpha}} = (\sigma^\mu\partial_\mu\overline{\lambda} = 0)_\alpha\ , \qquad D = 0$$

... and the Noether currents

$$\begin{aligned} j_\mu^{(5)} &= -\lambda\sigma_\mu\overline{\lambda}\\ j_{\mu\alpha} &= -i(F_{\mu\nu} + \tilde{F}_{\mu\nu})(\sigma^\nu\overline{\lambda})_\alpha - F_{\mu\nu}\partial^\nu\chi_\alpha\\ T_{\mu\nu} &= -\frac{1}{2}g_{\mu\nu}F^2 + F_{\mu\rho}\partial_\nu A^\rho + \frac{i}{2}\lambda\sigma_\mu(\partial_\nu\overline{\lambda}) - \frac{i}{2}(\partial_\nu\lambda)\sigma_\mu\overline{\lambda} + \frac{1}{2}g_{\mu\nu}D^2\end{aligned}$$

- Suitably improved, these currents can be embedded into the multiplet

$$J_{\alpha\dot{\alpha}} = 2W_\alpha\overline{W}_{\dot{\alpha}}$$

and the E.O.M. $D = 0$ implies that $\overline{D}^{\dot{\alpha}}J_{\alpha\dot{\alpha}} = 0$.

The Effect of Adding an FI Term

- Now we want to ask what contribution $\Xi_{\alpha\dot{\alpha}}$ arises from adding an FI term.

$$J_{\alpha\dot{\alpha}} = 2W_\alpha \overline{W}_{\dot{\alpha}} + \Xi_{\alpha\dot{\alpha}} \qquad \Xi_{\alpha\dot{\alpha}} = \ ?$$

ξ-dependent contribution, linear in the fields

$$\begin{aligned}\mathcal{L} &= \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot{\alpha}}\overline{W}^{\dot{\alpha}}|_{\overline{\theta\theta}}\right) + 2\xi V|_{\theta\theta\overline{\theta\theta}} \\ &= -\frac{1}{4}F^2 - \frac{i}{2}\lambda\sigma^\mu(\partial_\mu\overline{\lambda}) + \frac{i}{2}(\partial_\mu\lambda)\sigma^\mu\overline{\lambda} + \frac{1}{2}D^2 + \xi\left(D + \frac{1}{2}\Box C\right)\end{aligned}$$

- The E.O.M. for D and C are now $\quad D = -\xi\ , \quad C \text{ unconstrained}$

...and the extra FI contributions to the Noether currents are

$$\left.\begin{aligned}\Delta j_\mu^{(5)} &= 0 \\ \Delta j_{\mu\alpha} &= \xi(\sigma_\mu\overline{\lambda})_\alpha \\ \Delta T_{\mu\nu} &= \xi g_{\mu\nu} D\end{aligned}\right\}$$

These terms alone do not have the correct SUSY transformation properties to fill out a multiplet, even after improvement terms are added [Dienes & B.T., ’09].

- This result forces us toward a rather strange pair of alternative possibilities:

1 The FI supercurrent does not exist: $\Xi_{\alpha\dot{\alpha}}$ vanishes and $J_{\alpha\dot{\alpha}}$ only depends on ξ through E.O.M.

2 Additional contributions from some other source are required whenever an FI term is present, in order to ensure that $J_{\alpha\dot{\alpha}}$ has the correct transformation properties.

We shall show that, in fact, <u>both</u> of these possibilities are realized in supergravity, depending on the formalism used, and whether that formalism preserves R_5-invariance.

Compensator Formalisms: Recipes for Supergravity

From the kitchen of Stelle, West, Ferrara, van Nieuwenhuizen

Recipe for Supergravity coupled to matter

Ingredients

1 globally supersymmetric theory
2 tsp. conformal compensators
1½ cup covariant derivatives

Directions **Step 1**: Promote the globally-supersymmetric theory to a superconformal one by stirring in an appropriate set of compensator fields, modifying the action as required. Let stand 10 min, covered.

Step 2: Make the compensated, globally superconformal theory local by covariantizing derivatives and replacing flat superspace integration measures with curved ones.

Step 3: "Freeze" the compensators to constant values in order to break the Extraneous symmetries of the superconformal Group. Serve chilled.

Chiral Formalism

- We begin with a globally supersymmetric theory described by a Kähler potential K, superpotential W (we set the prepotential $f_{ab} = 1$ for simplicity).

$$\mathcal{L} = \int d^4\theta K + \left[\int d^2\theta (W + W^\alpha W_\alpha) + \text{h.c.} \right]$$

We expand $K = \sum_n K_n$

where K_n has Weyl weight n.

1. D-term Action:

$$\mathcal{L}_D = \int d^4\theta \left[\Sigma\overline{\Sigma} e^{\widetilde{K}/3M_P^2} \right]$$

where $\widetilde{K} = \sum_n \left(\frac{\Sigma\overline{\Sigma}}{3M_P^2} \right)^{-n/2} K_n$

- We introduce a chiral (superfield) compensator Σ and its conjugate $\overline{\Sigma}$.
- These fields can be used to render the D-term action superconformal.

Compensators

Field	w	R_5
Σ (chiral)	1	2/3
$\overline{\Sigma}$ (antichiral)	1	-2/3

2. F-term Action:

(n: mass dim. of X).

<u>Redefinition</u> of fields:

$$\Phi_i \equiv \left(\frac{\Sigma}{\sqrt{3}M_P}\right)\widetilde{\Phi}_i$$

<u>Prescription</u> for operators:

$$X \rightarrow \left(\frac{\Sigma}{\sqrt{3}M_P}\right)^n \widetilde{X}$$

- The new $\widetilde{\Phi}_i$ fields have $w = r_5 = 0$, due to rescaling by Σ.
- Now, one can construct a superconformally invariant F-term action.

$$\mathcal{L}_F = \int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P}\right)^3 \widetilde{W} + \text{h.c.}$$

- Here, $\widetilde{W}$ has the same form as the original superpotential, but with $\Phi_i \rightarrow \widetilde{\Phi}_i$.

3. Freezing the Fields

$$\Sigma \to \sqrt{3} M_P \qquad \overline{\Sigma} \to \sqrt{3} M_P$$

Breaks Weyl, R_5 invariance
(even if the original theory had them!)

After freezing...

(Note: I'm omitting terms from covariant derivatives that vanish as $M_P \to \infty$.)

$$\mathcal{L} \to \int d^4\theta \, 3M_P^2 \left[1 + \frac{K}{3M_P^2} + \frac{K^2}{18 M_P^4} + \ldots \right] + \left[\int d^2\theta (\widetilde{W} + W^\alpha W_\alpha) + \text{h.c.} \right]$$

$$\xrightarrow{M_P \to \infty} \quad \underbrace{\int d^4\theta K + \left[\int d^2\theta (\widetilde{W} + W^\alpha W_\alpha) + \text{h.c.} \right]}_{\text{Same form as our original Lagrangian}} + \underbrace{3 \int d^4\theta \; M_P^2}_{\text{Extra c.c. term}}$$

- In the $M_P \to \infty$ limit, we recover our original Lagrangian, except that the Φ_i have been replaced by $\widetilde{\Phi}_i$.

Currents in the Chiral Formalism

- Since R_5 is always broken in this formalism, the superfield conservation equation is:

$$\boxed{\overline{D}^{\dot{\alpha}} J_{\alpha\dot{\alpha}} = D_\alpha S}$$

- As usual, currents are calculated using the standard Noether method, but from the full, <u>unfrozen</u> theory, <u>including</u> the compensators.

- Thus there are two distinct contributions to the currents that make up $J_{\alpha\dot{\alpha}}$:

1 The Noether contribution from the fields of the original theory

(Similar to the contribution in the original theory, but differs due to the fact that Φ_i and $\widetilde{\Phi}_i$ have different R_5-charges.)

2 The Noether contribution from the fields in Σ and $\overline{\Sigma}$

(An entirely new contribution that does not arise in the flat-space theory.)

- For typical Kähler potential terms involving the matter fields, the sum of these two contributions is equal to the result from the original, uncompensated theory.

Result in uncompensated theory

$$J^{(C)}_{\alpha\dot\alpha} = J_{\alpha\dot\alpha} + \frac{1}{3}[D_\alpha, \overline{D}_{\dot\alpha}]\underbrace{(K - \Phi_i K_i)}_{\text{Typically Cancels}}$$

$$K_i \equiv \frac{\partial K}{\partial \Phi_i}$$

Result in chiral formalism

- However, for an FI term $2\xi V$, the situation is different. No new contribution to $j^{\mu\alpha}$ or $T^{\mu\nu}$ remains after the compensators are frozen, but $j^{(5)}_\mu$ gets a contribution:

$$j^{(5)}_\mu = -\frac{4}{3}\xi A_\mu - \frac{\xi^2}{18M_P^2}\chi\sigma_\mu\overline{\chi} + \mathcal{O}(M_P^{-6}) \xrightarrow{M_P \to \infty} -\frac{4}{3}\xi A_\mu$$

From before...

$$\begin{aligned}\Delta j_{\mu\alpha} &= \xi(\sigma_\mu\overline{\lambda})_\alpha \\ \Delta T_{\mu\nu} &= \xi g_{\mu\nu} D\end{aligned}$$

$$\Xi_{\alpha\dot\alpha} = \frac{2\xi}{3}[D_\alpha, \overline{D}_{\dot\alpha}]V$$

- Now, thanks to the compensator contribution, $\Xi_{\alpha\dot\alpha}$ has the correct multiplet structure. However, it is <u>not gauge-invariant</u> and <u>breaks R_5-symmetry</u>.

FI Terms and Symmetries in the Chiral Formalism

- Consider a globally-supersymmetric theory with a $U(1)$ gauge group $U(1)'_{FI}$, which has a nonzero FI term:

$$K = 2\xi V + K' \qquad \text{(}K'\text{ is the non-FI part of the Kähler potential).}$$

$$\boxed{\mathcal{L}_D = \int d^4\theta \left[\underbrace{\Sigma\overline{\Sigma}e^{-2\xi V/3M_P^2}}_{\text{Not gauge invariant as } V \to V + i(\Lambda_{\rm FI} + \overline{\Lambda}_{\rm FI})} e^{\widetilde{K}/3M_P^2}\right]}$$

- We can fix this by altering the theory and assigning $U(1)_{\rm FI}$ charge to Σ and $\overline{\Sigma}$.

$$\boxed{\begin{aligned} \Sigma &\to e^{2i\xi/3M_P^2\Lambda_{\rm FI}}\Sigma \\ \overline{\Sigma} &\to e^{2i\xi/3M_P^2\Lambda_{\rm FI}}\overline{\Sigma} \end{aligned}}$$

Now the D-term action is $U(1)_{\rm FI}$ guge invariant.

F-term Actions and Global R-Invariance

- Charging Σ and $\overline{\Sigma}$ under $U(1)_{\mathrm{FI}}$ remedies gauge-invariance issues in the D-term action, but it causes similar issues in the F-term action.

$$\int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P^2}\right)^3 \tilde{W} \xrightarrow[\text{transformation}]{U(1)_{\mathrm{FI}}\text{ gauge}} \int d^2\theta \left(\frac{\Sigma}{\sqrt{3}M_P^2}\right)^3 \tilde{W} e^{\overbrace{3i\Lambda_{\mathrm{FI}}-3Q_W\Lambda_{\mathrm{FI}}}^{\textbf{Must cancel!}}}$$

- To make this happen, I must be able to assign $U(1)_{\mathrm{FI}}$ charges to the $\widetilde{\Phi}_i$ so that $\widetilde{W}$ transforms homogeneously, with charge $Q_W = 3$.

- If my theory has a global R-symmetry, $\widetilde{W}$ transforms homogeneously under this symmetry, with charge $r_{\widetilde{W}} = 2$. I can exploit this to arrange the correct field charges:

Old $U(1)'_{\mathrm{FI}}$ charge

$$\boxed{Q_{\widetilde{\Phi}_i} = Q'_{\widetilde{\Phi}} + \frac{3}{2} r_{\widetilde{\Phi}_i}}$$

Global R-charge

If my theory does not have a global R-symmetry, no consistent charge assignment exists, and my theory cannot have an FI term.

However...

- Now freezing $\Sigma, \overline{\Sigma}$ breaks $U(1)_{\rm FI} \times$ [super-Weyl] down to a $U(1)$ subgroup $U(1)_A$ that combines $U(1)_{\rm FI}$ gauge transformations with super-Weyl rescalings.

$$\boxed{U(1)_{\rm SW} \times U(1)_{\rm FI} \quad \rightarrow \quad U(1)_A \;\equiv\; U(1)_{\rm FI} - \frac{\xi}{M_P^2} U(1)_{\rm SW}}$$

- Regular gauge transformations (of the FI gauge field A_μ) are combined with local R_5 rotations.

$U(1)_{\rm FI}$ gauge field

Local R_5 connection field

$$\boxed{A'_\mu \;\equiv\; \frac{1}{\sqrt{1+\xi^2/M_P^4}} \left(A_\mu - \frac{\xi}{M_P^2} b_\mu \right)}$$

- Consequently, all fields with nonzero R-charge (including the gravitino $\psi_{\mu\alpha}$ and all gauginos λ_α) acquire $U(1)_A$ charges in the full, SUGRA theory!

This way of dealing with FI terms is problematic for many reasons:

- Such charge shifts are inconsistent with Dirac quantization in the presence of magnetic monopoles [Witten, ’86].
- Anomaly cancellation is highly nontrivial [Chamseddine & Dreiner, ’95; Castano et al. ’96; Bineutry et al., ’04; Elvang et al., ’06; many others]
- Additional global symmetries of the compensated theory persist in the frozen theory, contradicting expectations about global symmetries in supergravity [Seiberg & Komargodski, ’09].

An Explicit Example

- Consider a toy theory with three chiral superfields charged under a $U(1)_{\mathrm{FI}}$ gauge group.
- We assume a canonical Kähler potential:

Field	$U(1)'_{\mathrm{FI}}$	R_5
Φ_1	$+1$	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
λ_α	0	1

$$K = \Phi_1^\dagger e^{-V}\Phi_1 + \Phi_2^\dagger e^{V}\Phi_2 + \Phi_3^\dagger\Phi_3 + 2\xi V$$

$$\underbrace{W = y_1\Phi_1\Phi_2\Phi_3 + y_2\Phi_3^3}_{\text{The most general renormalizable superpotential consistent with the symmetries of the theory}}$$

Comments:

- This model has a global R_5 symmetry under which all Φ_i have charge 2/3.
- The only term which breaks (global) Weyl invariance is the FI term; the superpotential is Weyl-invariant.

- Now we introduce the compensators and rescale the matter fields so that they transform trivially under $U(1)_{\mathrm{SW}}$.

Global

Local (Gauged)

Matter fields rescaled

Field	$U(1)'_{\mathrm{FI}}$	$U(1)_{\mathrm{FI}}$	$U(1)_{\mathrm{SW}}$:	R_5	Weyl
$\widetilde{\Phi}_1$	+1	$1-2\xi/3M_P^2$	0	0	0
$\widetilde{\Phi}_2$	-1	$-1-2\xi/3M_P^2$	0	0	0
$\widetilde{\Phi}_3$	0	$-2\xi/3M_P^2$	0	0	0
Σ	0	$2\xi/3M_P^2$	2/3	2/3	1
λ_α	0	0	$*$	1	3/2
$\psi_{\mu\alpha}$	0	0	$*$	1	3/2

- We can see that the theory contains an extra global symmetry. One can interpret this symmetry as either a global version of the $U(1)'_{\mathrm{FI}}$ gauge symmetry of the original theory…

- …or, alternatively, one can interpret it as an R-symmetry by taking linear combinations of R_5, $U(1)_{\mathrm{FI}}$, and $U(1)'_{\mathrm{FI}}$.

- We now freeze the compensators to obtain the full theory, coupled to Poincaré supergravity.

Frozen Theory:

Global $U(1)'_{\rm FI}$ symmetry unbroken!

Field	$U(1)'_{\rm FI}$	$U(1)_A$
$\widetilde{\Phi}_1$	+1	$1-2\xi/3M_P^2$
$\widetilde{\Phi}_2$	-1	$-1-2\xi/3M_P^2$
$\widetilde{\Phi}_3$	0	$-2\xi/3M_P^2$
λ_α	0	$-\xi/3M_P^2$
$\psi_{\mu\alpha}$	0	$-\xi/3M_P^2$

Gauginos and gravitino have $U(1)_A$ charge.

- The symmetries of the frozen theory are not those of the original theory: $U(1)'_{\rm FI}$ has been replaced by the "gauged R-symmetry" $U(1)_A$.

- The full supergravity theory also has an exact global symmetry. Alternatively, this can be taken to be the global $U(1)'_{\rm FI}$, or a global copy of the original R_5 symmetry (made from linear combinations of $U(1)'_{\rm FI}$ and $U(1)_A$).

This runs contrary to the folk theorem (proven in string theory!) that there are no continuous global symmetries in supergravity.

Thus we conclude that FI terms cannot be coupled to supergravity in the chiral formalism

Ways out?

1). Break $U(1)_{\rm FI}$ gauge invariance.

- There is nothing intrinsically wrong with effective FI terms generated in conjunction with field VEVs.

$$\mathcal{L} = \frac{1}{4}\left(W^\alpha W_\alpha|_{\theta\theta} + \overline{W}_{\dot\alpha}\overline{W}^{\dot\alpha}|_{\overline{\theta\theta}}\right) + \overbrace{m^2 V^2}^{\text{Mass Term}}|_{\theta\theta\overline{\theta\theta}} + 2\xi V|_{\theta\theta\overline{\theta\theta}}\ .$$

- The presence of a mass term for V changes the equations of motion. Now:

$$(\Box - m^2)\chi_\alpha = (\Box - m^2)A_\mu = (\Box - m^2)\lambda_\alpha = (\Box - m^2)D = 0$$

$$M = N = 0 \qquad\qquad (\Box - m^2)C - \xi = 0$$

- These equations truncate the fields of the supercurrent supermultiplet so that $\partial^\mu C_\mu = \partial^\mu j^{(5)}_\mu = 0$, and the R_5 current is conserved on shell.
- Furthermore, since $U(1)_{\rm FI}$ is broken, the gauge-invariance of the supercurrent is no longer a concern.

2).Consider alternative formalisms.

- Other compensator formalisms exist by means of which theories with particular symmetry properties may be coupled to supergravity.
- In fact, another formalism exists which, like the chiral formalism, is minimal (meaning that the SUGRA multiplet has the fewest possible auxiliary degrees of freedom), but unlike the chiral formalism, preserves R_5.

We now turn to explore this formalism in detail...

Linear Formalism

1. D-term Action:

$$\mathcal{L}_D = \int d^4\theta \left[L \ln\left(\frac{L}{\Sigma_L \overline{\Sigma}_L}\right) + L \frac{\widetilde{K}_L}{3M_P^2} \right]$$

where $$\widetilde{K}_L = \sum_n \left(\frac{L}{3M_P^2}\right)^{-n/2} K_n$$

Compensators

Field	w	R_5
L (linear)	2	0
Σ_L (chiral)	1	2/3
$\overline{\Sigma}_L$ (antichiral)	1	-2/3

- For any chiral superfield Ω: $$\int d^4\theta L\Omega = \partial_\mu[\ \ldots\ ]$$

Total Derivative

- This means the action is invariant under a new symmetry $U(1)_L$, under which

$$\Sigma_L \to e^{i\Lambda_L}\Sigma$$
$$\overline{\Sigma}_L \to e^{-i\overline{\Lambda}_L}\overline{\Sigma}_L$$
$$L, \Phi_i, V \to L, \Phi_i, V$$

$$\mathcal{L}_D \to \mathcal{L}_D + i\int d^4\theta L(\Lambda_L - \overline{\Lambda}_L) = \mathcal{L}_D + \partial_\mu[\ \ldots\ ]$$

This symmetry is a fundamental ingredient in the linear formalism.

2. F-term Action:

$$\mathcal{L}_F = \int d^2\theta W + \text{h.c.} \longrightarrow \textbf{?}$$

- L is linear and cannot compensate for chiral pieces of the action.
- Σ_L is prevented from compensating in W by the requirement of $U(1)_L$ invariance.

It follows that the linear formalism can <u>only</u> be used when the F-term action is <u>already</u> superconformally invariant.

- However, it can still be useful to define a new set of fields $\widetilde{\Phi}_L$, with $w = r_5 = 0$, through rescalings alone:

$$\Phi_i = \left(\frac{\Sigma_L}{\sqrt{3}M_P}\right)^{w_i} \widetilde{\Phi}_{Li}$$

Weyl weight of Φ_i

3. Freezing the Fields

$$\Sigma_L \to \sqrt{3}M_P \quad \overline{\Sigma}_L \to \sqrt{3}M_P$$

$$L \to 3M_P^2$$

Breaks $U(1)_L \times$ super-Weyl down to super-Weyl invariance.

Breaks super-Weyl invariance down to R_5.

- In the linear formalism, freezing the fields breaks Weyl invariance, special SUSY, etc. but leaves R_5-invariance intact.

After freezing...

(Again, I'm omitting covariant-derivative terms.)

$$\mathcal{L}_D \to \int d^4\theta \left[3M_P^2 \ln\left(\frac{3M_P^2}{3M_P^2}\right) + K\right] = \int d^4\theta K$$

- So once again, in the $M_P \to \infty$ limit, we recover the original, flat-space theory.

Supercurrents in the Linear Formalism

- Once again, we expect two contributions to the Noether currents:

1	The contribution from the fields of the original theory
2	The contribution from the fields in Σ and $\overline{\Sigma}$

- However, unlike in the chiral formalism, no additional compensator contribution to $j^{(5)}_\mu$ (or any other current) survives freezing:

$$j^{(5)}_\mu|_{\Sigma,\overline{\Sigma}} = \frac{2i}{3}\left(\phi^*_\Sigma \partial_\mu \phi_\Sigma - \phi_\Sigma \partial_\mu \phi^*_\Sigma\right) + \frac{1}{3}\psi_\Sigma \sigma_\mu \overline{\psi}_\Sigma \xrightarrow{\Sigma_L,\overline{\Sigma}_L \to \sqrt{3}M_P} 0$$

- The matter fields in the compensated theory are the same Φ_i as in the original theory, so their Noether-current contributions are also the same.
- Thus the supercurrent superfield is the same as in the uncompensated theory.

$$\Xi_{\alpha\dot{\alpha}} = 0 \qquad J^{(L)}_{\alpha\dot{\alpha}} = J_{\alpha\dot{\alpha}}$$

However, we can go even further:

- We now prove that $\Xi_{\alpha\dot{\alpha}} = 0$ in <u>any</u> theory or formalism in which R_5-invariance is preserved [Dienes, B.T., '09].

- The R_5-current conservation law $\partial^\mu j^5_\mu = \partial^\mu C_\mu = 0$ implies that the SUSY transformations of the component fields in J_μ must reduce to...

$$\begin{aligned} \delta_\epsilon C_\mu &= i\epsilon\chi_\mu - i\overline{\epsilon}\overline{\chi}_\mu \\ \delta_\epsilon \chi_{\mu\alpha} &= (\sigma^\nu\overline{\epsilon})_\alpha(\partial_\nu C_\mu + i\hat{T}_{\nu\mu}) \\ \delta_\epsilon \hat{T}_{\nu\mu} &= 2\overline{\epsilon\sigma}_{\nu\rho}\partial^\rho\overline{\chi}_\mu + 2\epsilon\sigma_{\nu\rho}\partial^\rho\chi_\mu \end{aligned}$$

with

$$M = N = 0$$
$$D = -\Box C$$
$$\lambda_{\mu\alpha} = -i(\sigma^\nu\partial_\nu\overline{\chi}_\mu)_\alpha$$

...in order for the SUSY algebra to close on the multiplet.

- This implies that...

J_μ is a linear multiplet

- In order to preserve R_5-invariance, the fields in J_μ <u>must</u> transform in this manner.

- Consider two successive SUSY transformations action on $\chi_{\mu\alpha}$, with parameters η and ϵ. Since $J^{\mu\alpha}$ is a linear multiplet...

$$\begin{aligned}\delta_\epsilon\delta_\eta\chi_{\mu\alpha} &= (\sigma^\nu\overline{\eta})_\alpha(\partial_\nu\delta_\epsilon C_\mu + i\delta_\epsilon\hat{T}_{\nu\mu}) \\ &= \underbrace{-2i(\epsilon\sigma^\nu\overline{\eta})(\partial_\nu\chi_{\mu\alpha}) + 2i(\overline{\epsilon\eta})(\sigma^\nu\partial_\nu\overline{\chi}_\mu)_\alpha}_{\text{Depends on } \overline{\eta}\text{, but not } \eta}\end{aligned}$$

- Now consider the FI contribution to $\chi_{\mu\alpha}$. It must be linear in the component fields of the $U(1)_{\mathrm{FI}}$ gauge multiplet V, so the most general form it could take would be

$$\chi^\mu_\alpha = X\,(\sigma^\mu\overline{\lambda})_\alpha + Y\,\partial^\mu\chi_\alpha + Z\,(\sigma^{\mu\nu}\partial_\nu\chi)_\alpha$$

Undetermined (complex) coefficients

- Now take the double SUSY variation of this $\chi_{\mu\alpha}$. The η-dependent contribution is

$$\delta_\epsilon\delta_\eta\chi^\mu_\alpha\Big|_\eta = \left[(Y g^{\mu\nu} + Z\sigma^{\mu\nu})\eta\right]_\alpha \left(2i\overline{\epsilon\sigma}^\rho\partial_\nu\partial_\rho\chi + 2\overline{\epsilon}\partial_\nu\overline{\lambda}\right)$$

Must vanish!

Thus $Y = Z = 0$.

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$$
\boxed{\begin{aligned}
\delta_\epsilon \delta_\eta \chi_{\mu\alpha} &= (\sigma^\nu \overline{\eta})_\alpha (\partial_\nu \delta_\epsilon C_\mu + i \delta_\epsilon \hat{T}_{\nu\mu}) \\
&= \underbrace{-2i(\epsilon \sigma^\nu \overline{\eta})(\partial_\nu \chi_{\mu\alpha}) + 2i(\overline{\epsilon}\overline{\eta})(\sigma^\nu \partial_\nu \overline{\chi}_\mu)_\alpha}_{\text{Depends on } \overline{\eta}\text{, but not } \eta}
\end{aligned}}
$$

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$$

- Now take the double SUSY variation of this $\chi_{\mu\alpha}$. The η-dependent contribution is

$$
\boxed{\delta_\epsilon \delta_\eta \chi^\mu_\alpha \Big|_\eta = \left[(Y g^{\mu\nu} + Z \sigma^{\mu\nu})\eta\right]_\alpha \left(2i\overline{\epsilon}\overline{\sigma}^\rho \partial_\nu \partial_\rho \chi + 2\overline{\epsilon}\partial_\nu \overline{\lambda}\right)}
$$

Must Vanish!

$$
\boxed{\text{Thus } Y = Z = 0.}
$$

So what about X?

Let's compare the $\overline{\epsilon}$-dependent part of the double-variation…

$$\delta_\epsilon \delta_\eta \overline{\lambda}^{\dot{\alpha}}\Big|_{\overline{\epsilon}} = 2iX(\sigma^\mu \overline{\sigma}^\nu \partial_\nu \lambda)_\alpha (\overline{\epsilon\eta})$$

…to the corresponding result for a linear (R_5-preserving) multiplet:

$$-2iX^*(\sigma^\nu \overline{\sigma}^\mu \partial_\nu \lambda)_\alpha (\overline{\epsilon\eta})$$

Clearly not equal!

Thus $X = Y = Z = 0$.

- It follows that $\chi_{\mu\alpha}$ must vanish, and therefore $\Xi_{\alpha\dot{\alpha}}$ must as well. In other words…

No FI contribution to the supercurrent exists in an R_5-symmetric theory!

Our Toy Theory Reexamined:

- Let's return to our toy theory and examine its symmetry structure as we couple it to SUGRA in the linear formalism.
- The compensated theory includes an extra (local) $U(1)_L$ symmetry, but no additional independent symmetries beyond this.

Original Theory:

Field	$U(1)'_{\mathrm{FI}}$	R_5
Φ_1	$+1$	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
λ_α	0	1

Compensated Theory:

The "extra" $U(1)$

	Field	$U(1)_{\mathrm{FI}}$	$U(1)_{\mathrm{SW}}$: R_5	Weyl	$U(1)_L$
Original Fields	Φ_1	+1	2/3	2/3 1	0
	Φ_2	-1	2/3	2/3 1	0
	Φ_3	0	2/3	2/3 1	0
Rescaled Fields	$\widetilde{\Phi}_{L1}$	$1-2\xi/3M_P^2$	0	0 0	+1
	$\widetilde{\Phi}_{L2}$	$-1-2\xi/3M_P^2$	0	0 0	+1
	$\widetilde{\Phi}_{L3}$	$-2\xi/3M_P^2$	0	0 0	+1
	Σ_L	$2\xi/3M_P^2$	2/3	2/3 1	-1
	L	0	$*$	0 2	0
	λ_α	0	$*$	1 3/2	0
	$\psi_{\mu\alpha}$	0	$*$	1 3/2	0

After Freezing

FI gauge symmetry (not R-type!)

Local R-symmetry

Frozen Theory:

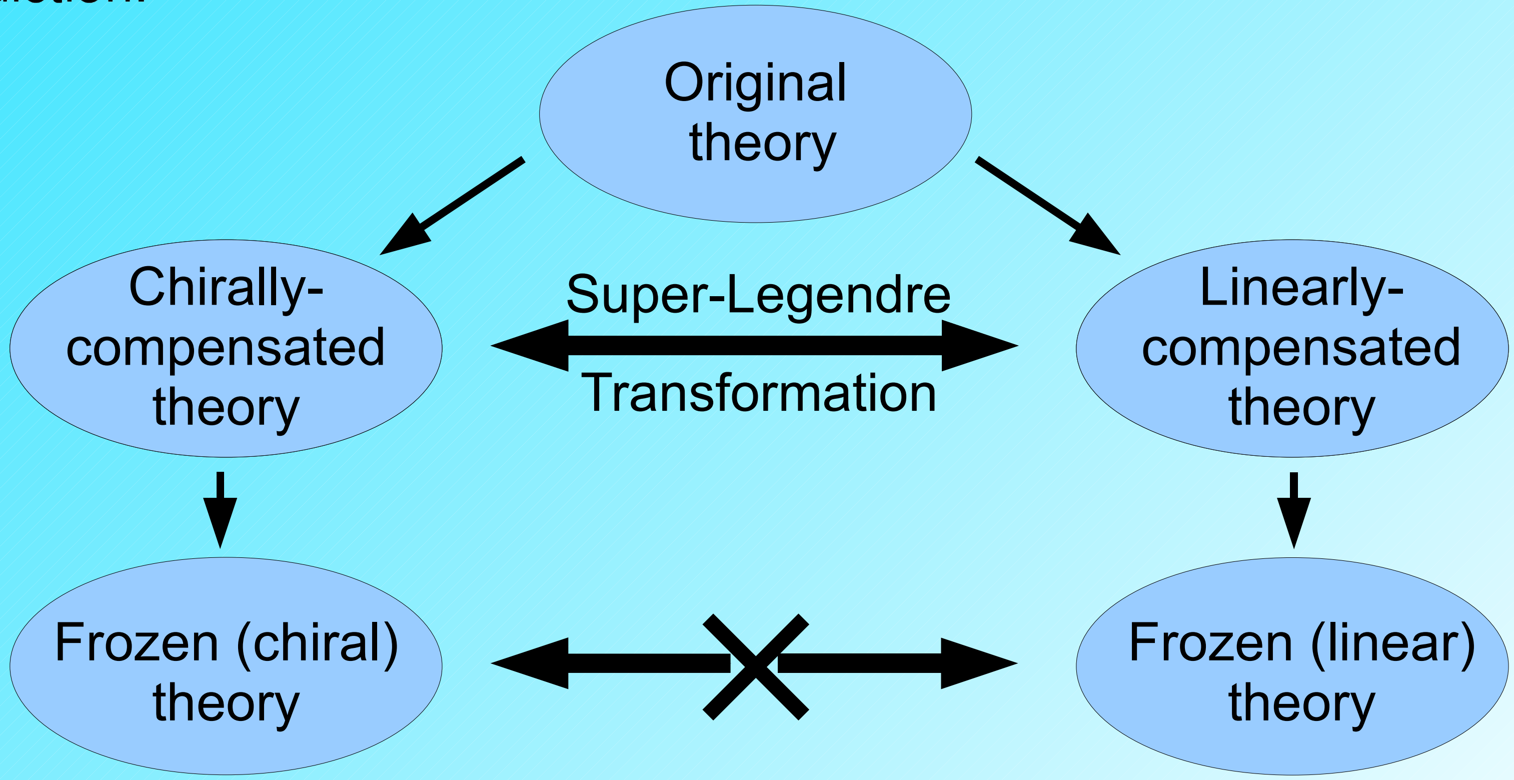
Field	$U(1)'_{\rm FI}$	R_G
Φ_1	+1	2/3
Φ_2	-1	2/3
Φ_3	0	2/3
$\widetilde{\Phi}_{L1}$	+1	2/3
$\widetilde{\Phi}_{L2}$	-1	2/3
$\widetilde{\Phi}_{L3}$	0	2/3
λ_α	0	1
$\psi_{\mu\alpha}$	0	1

- The FI gauge symmetry of the frozen theory is simply the $U(1)'_{\rm FI}$ symmetry of the original theory. It is not an R-type symmetry.
- Likewise, R_G is a local version of the R_5-symmetry of the original theory.
- Neither the gravitino nor the gauginos are charged under $U(1)'_{\rm FI}$.

The symmetries that remain intact are local.

Dualities between Formalisms

- The chiral and linear formalisms are known to be related by a duality transformation [Ferrara et al., ’83; Grisaru et al., ’84].
- More specifically, this transformation takes the form of a superfield-level Legendre transform, and is valid even in the presence of FI terms.
- However, this duality relationship exists at the level of the compensated theories, and certain equivalences are broken in the frozen theories; hence there is no contradiction.



Summary

- The Noether contributions to the currents $j_\mu^{(5)}$, $j_{\mu\alpha}$, and $T_{\mu\nu}$ that arise due to the presence of a nonzero FI term in a supersymmetric theory do not, in and of themselves, form a complete multiplet.

- For theories with broken R_5-symmetry, one can construct an FI contribution to $J_{\alpha\dot{\alpha}}$, but only by including additional contributions from the conformal compensators in the Noether calculation.

- For theories in which R_5-invariance is preserved, the additional FI contribution to the supercurrent superfield $\Xi_{\alpha\dot{\alpha}}$ must vanish, and $J_{\alpha\dot{\alpha}}$ can only depend on ξ through equations of motion.

Summary (continued)

- Moreover, in the chiral formalism, the presence of an FI term results in the presence of an exact global symmetry in the full supergravity theory. This theory also contains a gauged R-symmetry under which the gravitino and the gaguinos are charged. All of these result are highly problematic for FI terms.

- In the linear formalism, the symmetry content of the ‘full, supergravity theory is the same as it was in the original theory, aside from the fact that SUSY and R_5 are now local symmetries. All symmetries in the final, frozen theory are local.

Therefore

- If R_5-invariance is broken, either in the original theory itself or in the formalism used in coupling that theory to supergravity, fundamental FI terms are ruled out.

- If R_5-invariance is preserved by both original theory and supergravity formalism, then FI terms may be okay. (There are still highly nontrivial issues with anomaly-cancelation, maintaining R_5 invariance at the quantum level, etc.)

- In either case, effective FI terms that arise in conjunction with field VEVs that break $U(1)_{\mathrm{FI}}$ are perfectly consistent.

Thus the status of FI terms, which has had a tortu(r)ous history indeed, may look forward to a tortu(r)ous future as well.